



A stochastic model for forward–reverse logistics network design under risk

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ABSTRACT

Attention with reverse logistics networks has increased during the last decade since their economic impact has been increasingly important and as environmental legislation has been becoming stricter. In this paper, a *multi-period multi-echelon forward–reverse logistics network design under risk model* is developed. The proposed network structure consists of three echelons in the forward direction, (suppliers, facilities and distribution centers) and two echelons, in the reverse direction (disassembly, and redistribution centers), first customer zones in which the demands are stochastic and second customer zones in which the demand is assumed to be deterministic, but it may also assumed to be stochastic. The problem is formulated in a stochastic mixed integer linear programming (SMILP) decision making form as a multi-stage stochastic program. The objective is to maximize the total expected profit.

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1. Introduction

A reverse logistics network establishes a relationship between the market that releases used products and the market for “new” products. When these two markets coincide, then it is called a closed loop network, otherwise it is called an open loop network (Salema, Barbosa Póvoa, & Novais, 2007a). Very few optimization models for the design of supply chains with reverse flows are available in literature under uncertainty using scenarios to solve the model as uncertainty makes MILP models very hard to solve (Salema, Barbosa Póvoa, & Novais, 2005).

A facility location-allocation model for the collection, reprocessing and redistribution of carpet waste was presented by Louwers, Bert, Edo, Frans, and Simme Douwe (1999) to determine the locations and capacities of the regional recovery centers to minimize investment, processing, and transportation costs.

A generic stochastic model for the design of networks comprising both supply and return channels, organized in a closed loop system was presented by Listes (2002). This model considers one echelon forward network combined with two echelon reverse network. The uncertainty is handled in a stochastic formulation by means of discrete alternative scenarios.

A cost-minimization model for a multi-time-step, multi-type hazardous-waste reverse logistics system was presented by Hu, Sheu, and Huang (2002).

A stochastic programming model and solution algorithm for solving supply chain network design problems of a realistic scale was proposed by Santoso, Ahmed, Goetschalckx, and Shapiro (2005). The proposed solution methodology integrates a recently proposed sampling strategy, the sample average approximation scheme, with an accelerated Benders decomposition algorithm to quickly compute high quality solutions to large-scale stochastic supply chain design problems with a huge (potentially infinite) number of scenarios.

A strategic and tactical model for the design and planning of supply chains with reverse flows was proposed by Salema et al. (2007a). The authors considered the network design as a strategic decision, while tactical decisions are associated to production, storage and distribution planning.

A general reverse logistics location allocation model was developed by El Saadany and El-Kharbotly (2004) in a mixed integer linear programming (MILP) form. The model behavior and the effect of different reverse logistics variables on the economy of the system were studied. Demand in this proposed model is deterministic.

A carpet reverse logistics supply chain was simulated by Biehl, Edmund, and Matthew (2007) and used a designed experiment to analyze the impact of the system design factors as well as environmental factors impacting the operational performance of the reverse logistics system.

A stochastic approach to the case study of recycling sand from demolition waste was proposed by Listes and Dekker (2005). In this approach, the uncertainty is related to the demand sources and quality, i.e. from which locations the sand to be recycled is originated and its characteristics.

A MILP model for the design and planning of an integrated forward and reverse chain was proposed by Salema et al. (2005).

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Salema, Barbosa Póvoa, and Novais (2007b) studied the design of a reverse distribution network and found that most of the proposed models on the subject are case based and, for that reason, they lack generality. The model contemplates the design of a generic reverse logistics network where capacity limits, multi-product management and uncertainty on product demands and returns are considered. A mixed integer formulation is developed. This formulation allows for any number of products, establishing a network for each product while guaranteeing total capacities for each facility at a minimum cost. But the inventory was not taken into consideration. An illustrative case is presented, which allowed the model generality to be corroborated within very satisfactory computational times.

In present work, a multi period multi echelon forward–reverse logistics network model is developed for design purposes under risk. The problem is formulated in a stochastic mixed integer linear programming (SMILP) decision making form as a multi-stage stochastic program. The objective of the model is to maximize the total expected profit. Decisions are taken to determine the following:

- Suppliers, facilities, distribution centers, disassembly, and redistribution centers locations,
- Production at each location (what and how much to produce),
- Transported quantity of goods between locations, and
- Quantity of goods to hold as inventory at each period.

2. Model description

The model is a formulation for the forward–reverse logistics network design problem. The network is a multi-period multi-echelon, where it consists of suppliers, facilities, distributors, and first customers in the forward direction. In the reverse direction it consists of disassembly, disposal, redistribution locations and second customers, as shown in Fig. 1.

Costs incurred at different nodes are as follows:

(1) Suppliers:

These include investment fixed costs, materials costs, recycling costs, and transportation costs.

(2) Facilities:

These include investment fixed costs due to the opening of each facility, manufacturing costs, remanufacturing costs, non-utilized capacity costs, storage costs, and transportation costs.

(3) Distributors:

These include investment fixed costs due to the opening of each distributor, shortage costs, storage costs, and transportation costs.

(4) Disassembly locations:

These include investment fixed costs due to the operation of each disassembly location, returned price, disassembly, inspection and sorting costs, repairing costs, and transportation costs.

(5) Redistribution centers:

These include investment fixed costs due to the opening of each redistribution center, and transportation costs.

(6) Disposal locations:

These include investment fixed costs due to the opening of each disposal center, and disposal costs.

3. Model assumptions and limitations

The following are the assumptions considered in the present model:

1. The model is a multi-period.
2. Customers' locations are known and fixed with stochastic demands.
3. The returned quantities are stochastic and depend on the first customer demand.
4. The quality of remanufactured and repaired products is different from the new ones.
5. The potential locations of suppliers, facilities, distributors, disassemblies, and redistributors are known.
6. Costs parameters (fixed, material, manufacturing, non-utilized capacity, shortage, transportation, holding, recycling, remanufacturing, disassembly, and disposal costs) are known for each location and time period.
7. Capacity of each location is known for each time period.
8. The shortage cost depends on the shortage quantity and time.
9. The holding cost depends on the residual inventory at the end of each period.
10. Integer number of batches is transported.

4. Model formulation

The model involves the following sets, parameters and decision variables:

Sets:

- S: potential number of suppliers, indexed by s .
- F: potential number of facilities, indexed by f .
- D: potential number of distributors, indexed by d .
- C: potential number of first customers, indexed by c .
- A: potential number of disassembly locations, indexed by a .
- R: potential number of redistributors, indexed by r .
- P: potential number of disposal locations, indexed by p .
- K: potential number of second customers, indexed by k .
- T: number of periods, indexed by t .

Parameters:

- D_{ct} demand of first customer c in period t ,
- μ_{ct} demand mean of first customer c in period t ,
- σ_{ct} demand standard deviation of first customer c in period t ,
- D_{kt} demand of the second customer k in period t ,
- P_{ct} unit price at the first customer c in period t ,
- P_{kt} unit price at second customer k in period t ,

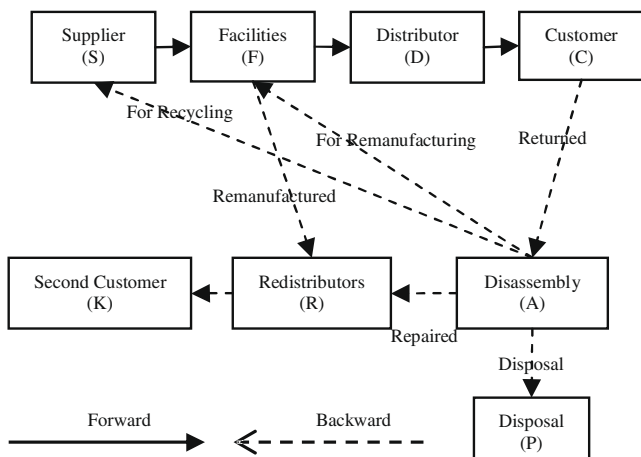


Fig. 1. The proposed forward–reverse logistics network.

- F_i fixed cost of opening location i .
- DS_{ij} distance between any two locations i and j .
- $DS_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$. where, x_i and y_i represent the Cartesian coordinates of location i .
- SC_{st} capacity of supplier s in period t ,
- SRC_{st} recycling capacity of supplier s in period t ,
- FC_{ft} manufacturing capacity in hours of facility f in period t ,
- RFC_{ft} remanufacturing capacity in hours of facility f in period t ,
- SC_{ft} storage capacity of facility f in period t ,
- DC_{dt} maximum capacity of distributor d in period t ,
- AC_{at} capacity of disassembly a in period t ,
- RC_{rt} capacity of redistributor r in period t ,
- PC_{pt} capacity of disposal p in period t ,
- MC_{st} material cost per unit supplied by supplier s in period t ,
- RC_{st} recycling cost per unit recycled by supplier s in period t ,
- FC_{ft} manufacturing cost per unit manufactured by facility f in period t ,
- RFC_{ft} remanufacturing cost per unit remanufactured by facility f in period t ,
- DAC_{at} disassembly cost per unit disassembled by disassembly location a in period t ,
- RPC_{at} repairing cost per unit repaired by disassembly location a in period t ,
- PC_{pt} disposal cost per unit disposed by disposal location p in period t ,
- N_{cf} non utilized manufacturing capacity cost per hour of facility f ,
- RN_{cf} non utilized remanufacturing cost per hour of facility f ,
- Sc : shortage cost per unit per period,
- Fh_f manufacturing time per unit in hours at facility f ,
- RF_{hf} remanufacturing time per unit in hours at facility f ,
- FH_f holding cost per unit per period at the store of facility f ,
- DH_d holding cost per unit per period at distributor d store,
- B_s, B_r, B_d, B_a & B_r : batch size from supplier s , facility f , distributor d , disassembly a , and, redistributor r respectively.
- Tc : transportation cost per unit per kilometer.
- RR : return ratio at the first customers.
- Rc : recycling ratio.
- Rm : remanufacturing ratio.
- Rc : repairing ratio.
- Rp : disposal ratio.

Decision variables:

- L_i binary variable equals 1 if location i is open and 0 otherwise.
- L_{ij} binary variable equals 1 if a transportation link is established between any two locations i and j .
- Q_{ijt} flow of batches from location i to location j in period t .
- I_{fft} flow of batches from facility f to its store in period t ,
- I_{fdt} flow of batches from store of facility f to distributor d in period t ,
- R_{ft} the residual inventory of the period t at store of facility f .
- R_{dt} the residual inventory of the period t at distributor d .

The relations between different nodes are shown in Fig. 2.

In the forward direction, suppliers are responsible for supplying of raw materials to facilities. Facilities are responsible for manufacturing of virgin products and supplying some of them to the distributors and storing the rest for the next periods; if it is profitable. Distributors are responsible for the distribution of new products to the first customers and/or storing some of them for the next periods, and customers' nodes may represent one customer, a retailer, or a group of customers and retailers.

In the reverse direction, the first customers return the used products to the disassembly locations. Disassembly locations are responsible for disassembling and sorting of the returned products to recyclable, remanufacturable, repairable, and dispos-

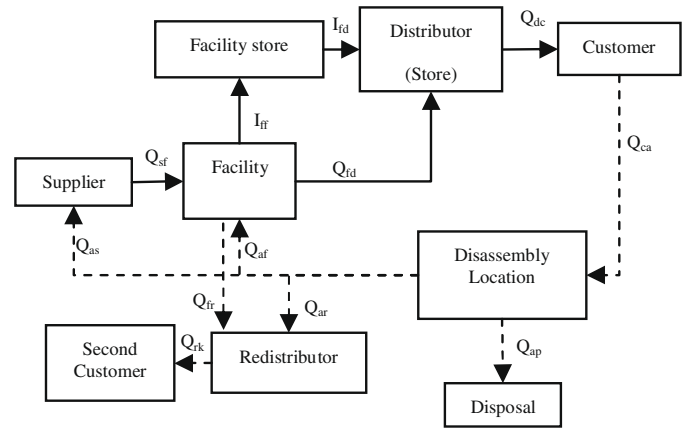


Fig. 2. Model flow.

able, and they are also responsible for supplying the recyclable to the suppliers, the remanufacturable to the facilities, the disposable to the disposal locations, and to repair the repairable products and supplying them directly to the redistribution locations. Suppliers are responsible for recycling of the returned products and supplying of recycled materials to facilities. Facilities are responsible for remanufacturing of used products and supplying them to the redistribution locations. Disposal locations are responsible for disposing of disposable products. Redistribution locations are responsible for the distribution of refurbished products to the second customers.

4.1. Objective function

The objective of the model is to maximize the total expected profit of the forward–reverse network.

Total expected profit = total expected income – total expected cost.

4.1.1. Total expected income

Total expected income = first sales + second sales.

$$\text{First sales} = \sum_{d \in D} \sum_{c \in C} \sum_{t \in T} Q_{dct} B_d P_{ct} \tag{1}$$

$$\text{Second sales} = \sum_{r \in R} \sum_{k \in K} \sum_{t \in T} Q_{rkt} B_r P_{kt} \tag{2}$$

4.1.2. Total expected cost

Total expected cost = fixed costs + material costs + manufacturing costs + non-utilized capacity costs + shortage costs + purchasing costs + disassembly costs + recycling profit + remanufacturing cost + repairing cost + disposal cost + transportation costs + inventory holding costs.

The costs are as follows:

(1) Fixed costs

$$\sum_{s \in S} F_s L_s + \sum_{f \in F} F_f L_f + \sum_{d \in D} F_d L_d + \sum_{a \in A} F_a L_a + \sum_{r \in R} F_r L_r + \sum_{p \in P} F_p L_p \tag{3}$$

(2) Material cost

$$\sum_{s \in S} \sum_{f \in F} \sum_{t \in T} Q_{sft} B_s MC_{st} - \sum_{a \in A} \sum_{s \in S} \sum_{t \in T} Q_{ast} B_a (MC_{st} - RC_{st}) \tag{4}$$

(3) Manufacturing costs

$$\sum_{f \in F} \sum_{d \in D} \sum_{t \in T} Q_{fdt} B_f FC_{ft} + \sum_{f \in F} \sum_{d \in D} \sum_{t \in T} I_{fdt} B_f FC_{ft} \tag{5}$$

(4) Non-utilized capacity cost (for facilities)

$$\sum_{f \in F} \left(\sum_{t \in T} \left((FC_{ft}/Fh_f)L_f - \sum_{d \in D} (Q_{fdt}B_f) - \sum_{d \in D} (I_{fdt}B_f) \right) NC_f \right) + \sum_{f \in F} \left(\sum_{t \in T} \left((RFC_{ft}/RFh_f)L_f - \sum_{r \in R} (Q_{fdt}B_f) \right) RNC_f \right) \quad (6)$$

(5) Shortage cost (for distributor)

$$\left(\sum_{c \in C} \left(\sum_{t \in T} \left(\sum_{t=1}^t D_{ct} - \sum_{t=1}^t \sum_{d \in D} Q_{dct}B_d \right) \right) \right) SC \quad (7)$$

(6) Purchasing costs

$$\sum_{c \in C} \sum_{a \in A} \sum_{t \in T} Q_{cat}P_cB_cQL_c \quad (8)$$

(7) Disassembly costs

$$\sum_{c \in C} \sum_{a \in A} \sum_{t \in T} Q_{cat}B_cDA_{cat} \quad (9)$$

(8) Recycling costs

$$\sum_{a \in A} \sum_{s \in S} \sum_{t \in T} Q_{ast}B_aRC_{st} \quad (10)$$

(9) Remanufacturing costs

$$\sum_{r \in F} \sum_{r \in R} \sum_{t \in T} Q_{ftr}B_rRFC_{ft} \quad (11)$$

(10) Repairing costs

$$\sum_{a \in A} \sum_{r \in R} \sum_{t \in T} Q_{art}B_aRPC_{at} \quad (12)$$

(11) Disposal costs

$$\sum_{a \in A} \sum_{p \in P} \sum_{t \in T} Q_{apt}B_aPC_{pt} \quad (13)$$

(12) Transportation costs

$$\sum_{t \in T} \sum_{s \in S} \sum_{f \in F} Q_{sft}B_sTcDS_{sf} + \sum_{t \in T} \sum_{f \in F} \sum_{d \in D} Q_{fdt}B_fTcDS_{fd} + \sum_{t \in T} \sum_{a \in A} \sum_{s \in S} Q_{ast}B_aTcDS_{as} + \sum_{t \in T} \sum_{a \in A} \sum_{f \in F} Q_{aft}B_aTcDS_{af} + \sum_{t \in T} \sum_{a \in A} \sum_{p \in P} Q_{apt}B_aTcDS_{ap} + \sum_{t \in T} \sum_{a \in A} \sum_{r \in R} Q_{art}B_aTcDS_{ar} + \sum_{t \in T} \sum_{f \in F} \sum_{r \in R} Q_{ftr}B_rTcDS_{fr} + \sum_{t \in T} \sum_{r \in R} \sum_{k \in K} Q_{rkt}B_rTcDS_{rk} \quad (14)$$

(13) Inventory holding costs

$$\sum_{f \in F} \sum_{t \in T} R_{ft}FH_f + \sum_{d \in D} \sum_{t \in T} R_{dt}DH_d \quad (15)$$

4.2. Constraints

This section is a representation to the constraints of the model:

4.2.1. Balance constraints

$$\sum_{s \in S} Q_{sft}B_s = \sum_{d \in D} Q_{fdt}B_f + I_{ft}B_f, \quad \forall t \in T, \quad \forall f \in F \quad (16)$$

$$I_{ft}B_f + R_f(t-1) = R_{ft} + \sum_{d \in D} Q_{fdt}B_f, \quad \forall t \in T, \quad \forall f \in F \quad (17)$$

$$\sum_{f \in F} (Q_{fdt} + I_{fdt})B_f + R_d(t-1) = R_{dt} + \sum_{c \in C} Q_{dct}B_d, \quad \forall t \in T, \quad \forall d \in D \quad (18)$$

$$\sum_{d \in D} Q_{dct}B_d \leq D_{ct} + \sum_{1}^t D_{ct}(t-1) - \sum_{d \in D} Q_{dc(t-1)}B_d, \quad \forall t \in T, \quad \forall c \in C \quad (19)$$

$$\sum_{a \in A} Q_{cat}B_c \leq \left(\sum_{d \in D} Q_{dct}B_d \right) RR, \quad \forall t \in T, \quad \forall c \in C \quad (20)$$

$$\sum_{c \in C} Q_{cat}B_c = \sum_{s \in S} (Q_{ast}B_a) + \sum_{f \in F} (Q_{aft}B_a) + \sum_{r \in R} (Q_{art}B_a) + \sum_{p \in P} (Q_{apt}B_a), \quad \forall t \in T, \quad \forall a \in A \quad (21)$$

$$\sum_{c \in C} (Q_{cat}B_c)RC = \sum_{s \in S} (Q_{ast}B_a), \quad \forall t \in T, \quad \forall a \in A \quad (22)$$

$$\sum_{c \in C} (Q_{cat}B_c)Rm = \sum_{f \in F} (Q_{aft}B_a), \quad \forall t \in T, \quad \forall a \in A \quad (23)$$

$$\sum_{c \in C} (Q_{cat}B_c)Rr = \sum_{r \in R} (Q_{art}B_a), \quad \forall t \in T, \quad \forall a \in A \quad (24)$$

$$\sum_{c \in C} (Q_{cat}B_c)Rd = \sum_{p \in P} (Q_{apt}B_a), \quad \forall t \in T, \quad \forall a \in A \quad (25)$$

$$\sum_{a \in A} (Q_{aft}B_a) = \sum_{r \in R} (Q_{ftr}B_r), \quad \forall t \in T, \quad \forall f \in F \quad (26)$$

$$\sum_{a \in A} (Q_{art}B_a) + \sum_{f \in F} (Q_{ftr}B_r) = \sum_{k \in K} (Q_{rkt}B_r), \quad \forall t \in T, \quad \forall r \in R \quad (27)$$

$$\sum_{r \in R} (Q_{rkt}B_r) \leq D_{kt}, \quad \forall t \in T, \quad \forall k \in K \quad (28)$$

Constraint (16) insures that, at each period, the flow entering to each facility from all suppliers is equal to the sum of the exiting from this facility to each facility and distributor stores.

Constraint (17) insures that the sum of the flow entering each facility store and its residual inventory from the previous period is equal to the sum of the exiting to each distributor and the residual inventory of the existing period.

Constraint (18) insures that the sum of the flow entering each distributor store from each facility or facility store and its residual inventory from the previous period is equal to the sum of the exiting to each customer and the residual inventory of the existing period.

Constraint (19) insures that the sum of the flow entering to each customer does not exceed the sum of the existing period demand and the previous accumulated back orders.

Constraint (20) insures that, at each period, the flow exiting from each customer to all disassembly locations does not exceed the sum of the entering to each customer.

Constraint (21) insures that, at each period, the flow entering each disassembly location from all customers is equal to the sum of flow exiting to each supplier for recycling, to each facility for remanufacturing, to each redistributor (repaired), and to each disposal location for disposing.

Constraint (22) insures that, at each period, the flow exiting from disassembly location to all suppliers to be recycled is equal to the entering to each disassembly location from all customers multiplied by the recycling ratio.

Constraint (23) insures that the flow exiting from disassembly location to all facilities to be remanufactured is equal to the entering to each disassembly location from all customers multiplied by the remanufacturing ratio.

Constraint (24) insures that the repaired flow exiting from disassembly location to all redistributors locations to be redistributed is equal to the entering to each disassembly location from all customers multiplied by the repairing ratio.

Constraint (25) insures that the flow exiting from disassembly location to all disposal locations to be disposed is equal to the entering to each disassembly location from all customers multiplied by the disposing ratio.

Constraint (26) insures that the sum of the remanufactured flow entering to each facility from each disassembly location is equal to the sum of the exiting to each redistributor location.

Constraint (27) insures that sum the of remanufactured flow entering to each redistributor location from all facilities and the repaired flow entering to it from all the disassembly locations is equal to the sum of flow exiting to each second customer.

Constraint (28) insures that flow entering to each second customer from all redistributors does not exceed the second customer demand at each period.

4.2.2. Capacity constraints:

$$\sum_{f \in F} Q_{sft} B_s \leq SC_{st} L_s, \quad \forall t \in T, \quad \forall s \in S \tag{29}$$

$$\left(\sum_{d \in D} Q_{fdt} B_f + \sum_{d \in D} I_{fnt} B_f \right) M H_f \leq FC_{ft} L_f, \quad \forall t \in T, \quad \forall f \in F \tag{30}$$

$$R_{ft} \leq SC_{ft} L_f, \quad \forall t \in T, \quad \forall f \in F \tag{31}$$

$$\sum_{f \in F} (Q_{fdt} + I_{fdt}) B_f + R_{dt-1} \leq DC_{dt} L_d, \quad \forall t \in T, \quad \forall d \in D \tag{32}$$

$$\sum_{s \in S} Q_{ast} B_a + \sum_{f \in F} Q_{aft} B_a + \sum_{r \in R} Q_{art} B_a + \sum_{p \in P} Q_{apt} B_a \leq PC_{pt}, \quad \forall t \in T, \quad \forall a \in A \tag{33}$$

$$\sum_{k \in K} Q_{rkt} B_r \leq RC_{rt}, \quad \forall t \in T, \quad \forall r \in R \tag{34}$$

$$\sum_{a \in A} Q_{ast} B_a \leq SRC_{st}, \quad \forall t \in T, \quad \forall s \in S \tag{35}$$

$$\sum_{a \in A} Q_{apt} B_a \leq PC_{pt}, \quad \forall t \in T, \quad \forall p \in P \tag{36}$$

Constraint (29) insures that, at each period, the sum of the flow exiting from each supplier to all facilities does not exceed the supplier capacity.

Constraint (30) insures that, at each period, the sum of the flow exiting from each facility to all facilities' stores and to all distributors does not exceed the facility capacity.

Constraint (31) insures that the residual inventory at each facility store does not exceed its storing capacity at each period.

Constraint (32) insures that, at each period, the sum of the residual inventory at each distributor from the previous period and the flow entering at the existing period from the facilities and facilities stores does not exceed this distributor capacity.

Constraint (33) insures that, at each period, the sum of the flow exiting from each disassembly location to all suppliers, facilities, redistributors and disposal locations does not exceed this disassembly location capacity.

Constraint (34) insures that, at each period, the flow exiting from each redistributor to the second customers does not exceed this redistributor capacity.

Constraint (35) insures that, at each period, the flow entering each supplier from all disassembly location does not exceed this supplier recycling capacity.

Constraint (36) insures that, at each period, the flow entering to each disposal location from all disassembly location does not exceed this disposing capacity.

4.2.3. Linking (contracts)-shipping constraints

$$Li_{sf} \leq \sum_{t \in T} Q_{sft}, \quad \forall s \in S, \quad \forall f \in F \tag{37}$$

$$Li_{fd} \leq \sum_{t \in T} (Q_{fdt} + I_{fdt}), \quad \forall f \in F, \quad \forall d \in D \tag{38}$$

$$Li_{dc} \leq \sum_{t \in T} Q_{dct}, \quad \forall d \in D, \quad \forall c \in C \tag{39}$$

$$Li_{ca} \leq \sum_{t \in T} Q_{cat}, \quad \forall a \in A, \quad \forall c \in C \tag{40}$$

$$Li_{as} \leq \sum_{t \in T} Q_{ast}, \quad \forall s \in S, \quad \forall a \in A \tag{41}$$

$$Li_{af} \leq \sum_{t \in T} Q_{aft}, \quad \forall f \in F, \quad \forall a \in A \tag{42}$$

$$Li_{ar} \leq \sum_{t \in T} Q_{art}, \quad \forall r \in R, \quad \forall a \in A \tag{43}$$

$$Li_{ap} \leq \sum_{t \in T} Q_{apt}, \quad \forall p \in P, \quad \forall a \in A \tag{44}$$

$$Li_{fr} \leq \sum_{t \in T} Q_{frr}, \quad \forall r \in R, \quad \forall f \in F \tag{45}$$

$$Li_{rk} \leq \sum_{t \in T} Q_{rkt}, \quad \forall k \in K, \quad \forall r \in R \tag{46}$$

Constraints (37)–(46) insure that there are no links between any locations without actual shipments during all periods.

4.2.4. Shipping-linking constraints

$$\sum_{t \in T} Q_{sft} \leq M Li_{sf}, \quad \forall f \in F, \quad \forall s \in S \tag{47}$$

$$\sum_{t \in T} (Q_{dct} + I_{fdt}) \leq M Li_{fd}, \quad \forall f \in F, \quad \forall d \in D \tag{48}$$

$$\sum_{t \in T} Q_{dct} \leq M Li_{dc}, \quad \forall d \in D, \quad \forall c \in C \tag{49}$$

$$\sum_{t \in T} Q_{cat} \leq M Li_{ca}, \quad \forall a \in A, \quad \forall c \in C \tag{50}$$

$$\sum_{t \in T} Q_{ast} \leq M Li_{as}, \quad \forall s \in S, \quad \forall a \in A \tag{51}$$

$$\sum_{t \in T} Q_{aft} \leq M Li_{af}, \quad \forall f \in F, \quad \forall a \in A \tag{52}$$

$$\sum_{t \in T} Q_{art} \leq M Li_{ar}, \quad \forall r \in R, \quad \forall a \in A \tag{53}$$

$$\sum_{t \in T} Q_{apt} \leq M Li_{ap}, \quad \forall p \in P, \quad \forall a \in A \tag{54}$$

$$\sum_{t \in T} Q_{frr} \leq M Li_{fr}, \quad \forall r \in R, \quad \forall f \in F \tag{55}$$

$$\sum_{t \in T} Q_{rkt} \leq M Li_{rk}, \quad \forall r \in R, \quad \forall k \in K \tag{56}$$

Constraints (47)–(56) ensure that there is no shipping between any non linked locations.

4.2.5. Maximum number of activated locations constraints

$$\sum_{s \in S} L_s \leq S \tag{57}$$

$$\sum_{f \in F} L_f \leq F \tag{58}$$

$$\sum_{d \in D} L_d \leq D \tag{59}$$

$$\sum_{a \in A} L_a \leq A \tag{60}$$

$$\sum_{r \in R} L_r \leq R \tag{61}$$

$$\sum_{p \in P} L_p \leq P \tag{62}$$

Constraints (57)–(62) limit the number of activated locations, where the sum of binary decision variables which indicate the number of activated locations, is less than the maximum limit of activated locations (taken equal to the potential number of locations).

5. Results and discussions

The behavior of the model has been studied with different model parameter. Demand mean and return ratio are taken to represent the main affecting parameters in the present study. Other

parameters are assumed to be constant and having the values given in Table 1. Demand mean varies between 0 and 3000 units per period for each customer. Therefore, the total demand means ranging from 0 to 12,000 units for all customers. The maximum capacity of the network equals 10,500 units per period which is limited by the maximum capacity of facilities (in present case the maximum capacity of any facility in the network is limited by 3500 units per period). Return ratio varies between 0% and 100% of the first used products.

Table 1
Nominal values of the model parameters.

Parameter	Value
Virgin unit price	100%
First customer demand mean per period (units)	500 and 1000
Increasing rate of the first customer demand mean	1
Second good's price	80%
First customer demand standard deviation (units)	50
Max return ratio	60%
Holding, shortage and non-utilized capacity cost per unit perperiod	10%
Material and manufacturing costs per unit	20%
Disassembly cost per unit	3%
Recycling cost per unit	5%
Remanufacturing cost per unit	10%
Repairing cost	5%
Disposal cost	1%
Max number of operating suppliers, facilities, distributors, disassemblies and redistributors	3
Max number of first customers	4
Max number of second customers	2
Supplier and distributor locations fixed costs	20,000
Facility location fixed costs	50,000
Disassembly location fixed costs	15,000
Redistribution location fixed costs	10,000
Disposal location Fixed costs	5000
Supplier and distributor capacity (units)	4000
Supplier recycling capacity (units)	2000
Facility manufacturing capacity (h)	7000
Facility remanufacturing capacity (h)	2000
Manufacturing and remanufacturing hours per unit	2
Facility's storing capacity per period (units)	1000
Disassembly location capacity (units)	3000
Redistribution location capacity (units)	3000
Disposal location capacity (units)	1000
Transportation cost per kilometer per unit	0.001
Second customer demand	2000

Initial experimentations showed that increasing the number of scenarios increases drastically the processing time with limited benefit in solution accuracy. The difference in results did not exceed 0.1% when increasing the number of scenarios from 27 to 125. Therefore, only 27 scenarios were chosen in the present work for three periods for a normally distributed discretized demand over three intervals.

The model is built by using Mosel language, which can be work as both a modeling language and a programming language (www.dashoptimization.com). Following this model language, the demand distribution can take any form since it accepts a large number of function forms.

All calculations were carried out using XpressSP 2006a on a 1.73 GHz Intel Centrino processor and 512MB RAM PC.

5.1. Effect of demand mean

In general, the increase in demand mean increases the total expected profit as shown in Fig. 3. The total expected profit is linearly proportional to the total demand. At certain instances, it decreases slightly due to the shortage cost as it is not profitable to open an extra location. At certain demand mean, it is profitable to open another location to fulfill the new demand. The same behavior continues with the increase in demand mean until the total demand exceeds the maximum permissible capacity of the network and it is not possible to open extra locations.

5.2. Effect of return ratio

The effect of return ratio is studied for demand means of 500 and 1000 as a sample of representing the cyclic behavior of the model as depicted from Fig. 3.

In general, the increase in return ratio increases the total expected profit. Fig. 4 shows the relation between the total expected profit and return ratio. It is evident that increasing the return ratio increases the total expected profit. At demand mean equals 500 units per customer per period, the total expected profit does not change with the increase in the return ratio from 0% to 20%, since, it is not profitable to open any reverse line. The resulted optimal network is as shown in Fig. 5.

After this range, the increase in the return ratio increases the total expected profit and a reverse line is opened. The resulted optimal network is as shown in Fig. 6.

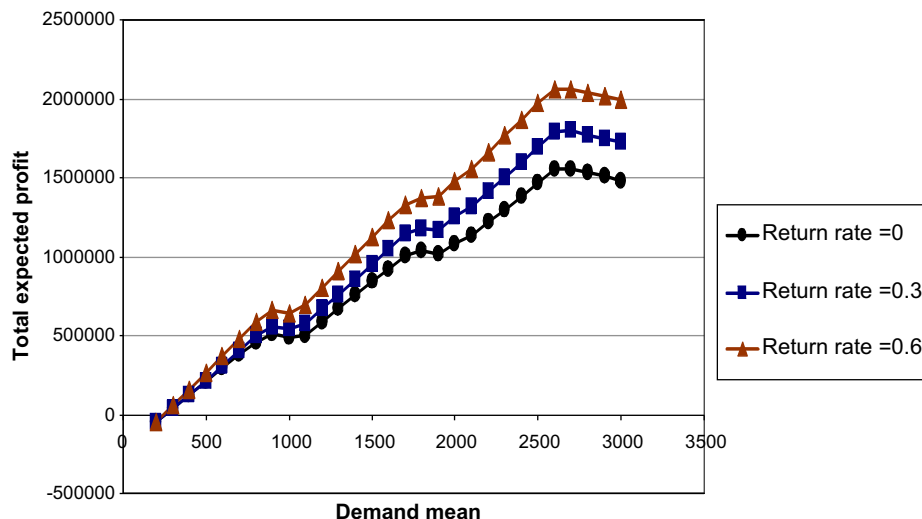


Fig. 3. Total expected profit vs. demand mean per customer per period.

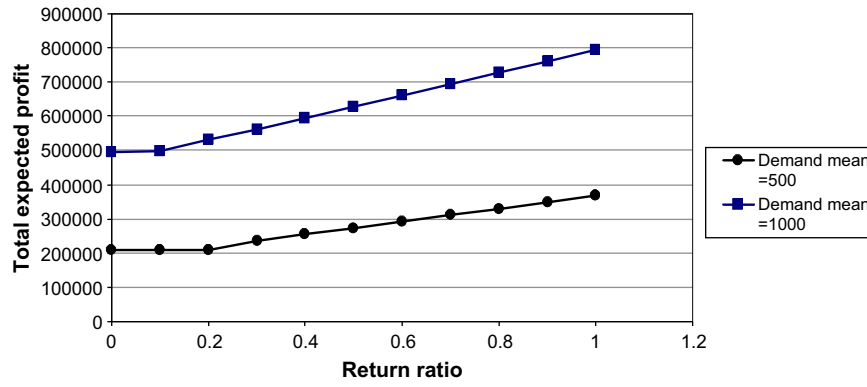


Fig. 4. Total expected profit % vs. return ratio per period.

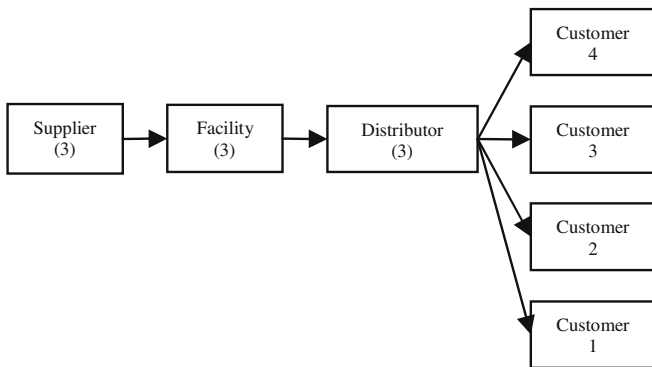


Fig. 5. Optimal network design # 1.

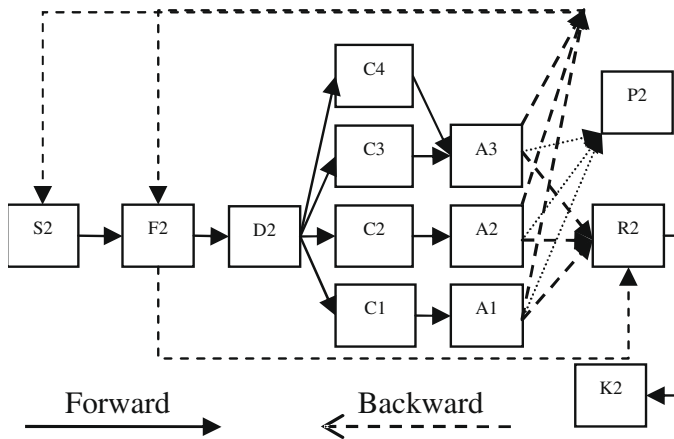


Fig. 6. Optimal network design # 2.

At demand mean equals 1000 units per customer per period, the total expected profit does not change with the increase in the return ratio from 0% to 10%, since, it is not profitable to open any reverse line, the resulted optimal network is also as shown in Fig. 5. After this range, the increase in the return ratio increases the total expected profit, since, the reverse line is opened, and the resulted optimal network is as shown in Fig. 6.

From Fig. 4, it can be seen that, the increase in the total expected profit for demand mean of 1000 units per customer per period starts before the increase for demand mean of 500, since, opening of the reverse line depends on the returned quantities not only on the return ratio. The increasing rate of the total expected profit with the return ratio for demand mean of 1000 units

per customer per period is bigger than the increasing rate for demand mean of 500, since, the increase in the total expected profit depends on the returned quantities not only on the return ratio.

6. Example

An example of the network given in Fig. 1 for ten periods was solved using the parameters values given in Table 1 to verify the developed model. To simplify the analysis of the results, the demand is assumed to be deterministic. Identical demand pattern of all customers is assumed for all planning periods as shown in Fig. 7. The demand increases during the first four periods, decreases in the following three periods, and increases again during the last three periods.

The optimal network is shown in Fig. 8. The maximum capacity of this network is limited by the capacity of the facility which is 3500 units per period.

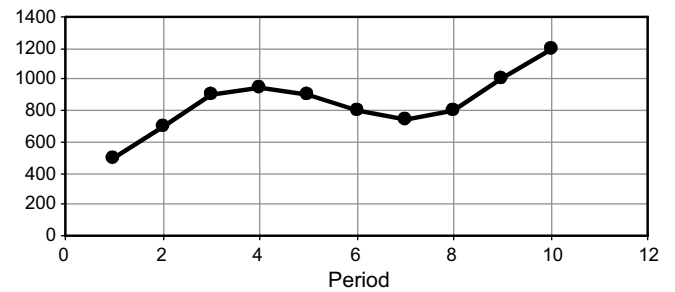


Fig. 7. Demand pattern.

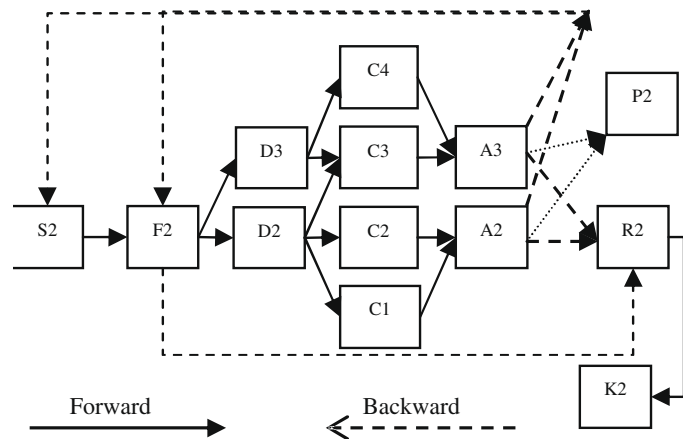


Fig. 8. Optimal network design of the example.

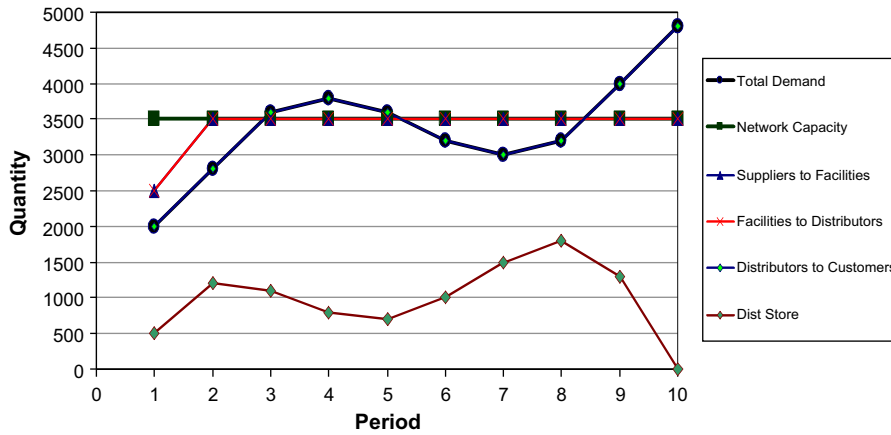


Fig. 9. Optimal flows between forward locations.

The optimal flows between the forward locations are as shown in Fig. 9. The maximum profit of this network equals 2360,716 as much as unit selling price.

It can be seen that the demand during the first and second periods is less than the network capacity while during the third, fourth and fifth period it is higher than the maximum available capacity. The optimal solution showed that the facility will produce and

store during the first two periods for the use during the coming periods. The same behavior is evident for the ninth and the tenth periods, where the capacity is less than the demand while it is higher during the preceding periods.

It is also evident from Fig. 9 that there is no inventory at the facility (the two lines representing the input to the facility and the output from facility to distributors are coincide with each

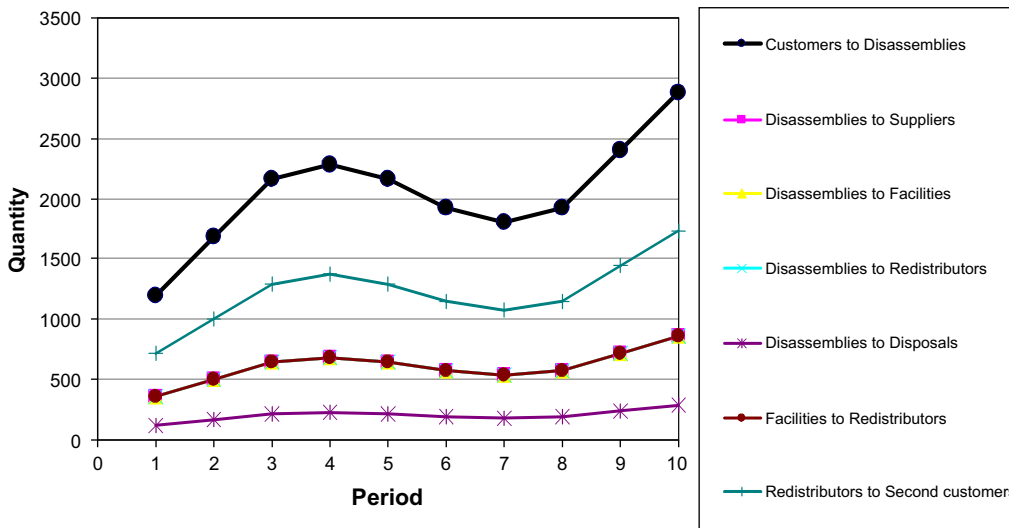


Fig. 10. The resulted optimal flows between reverse locations.

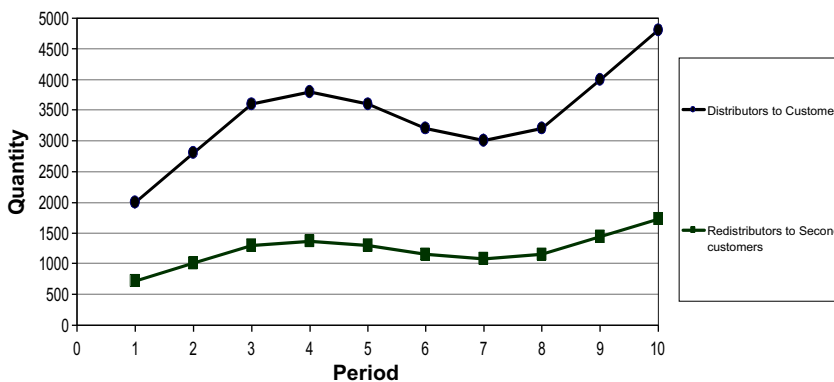


Fig. 11. The resulted satisfied demand of the first and the second customers.

other) and all inventories are only at the distributors, which is represented by the lowest curve.

The optimal flows between the reverse locations are as shown in Fig. 10. The flow of materials from disassembly locations to supplier, facility and redistributor are identical due to the assumed fixed return ratio to three destinations (0.3 of the return quantity).

The actual quantities flowing to the first and the second customers are of identical pattern due to the assumed fixed maximum return ratio as shown in Fig. 11.

The results in Figs. 9–11 verify the application of the developed model.

7. Conclusion

From the previous study, the following conclusions can be derived.

The proposed model is successful in designing forward–reverse logistics networks while considering multi-period stochastic demand with three echelons (suppliers, facilities and distributors) in the forward direction and two echelons (disassemblies and redistributors) in the reverse direction. It can only be used for single item, single product problems. The model is flexible to solve larger problems; however, it requires more powerful hardware since the number of scenarios increases exponentially with the increase of the number of periods.

The application of the proposed model showed that the total expected profit is directly affected by demand mean and return ratio for a given capacity of the network. Only integer number of batches can be transported during a period which limits the application of the model.

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